Cambridge Pre-U

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics 1 **May/June 2022**

2 hours

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

INSTRUCTIONS

- Answer **all** questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do **not** use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- At the end of the examination, fasten all your work together. Do **not** use staples, paper clips or glue.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document has **4** pages. Any blank pages are indicated.

1 A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 1$ and $u_{n+1} = \dfrac{4}{2 - u_n}$ for $n \geqslant 1$.

(a) Write down the values of u_2, u_3 and u_4. [2]

(b) Find $\displaystyle\sum_{n=1}^{50} u_n$. [3]

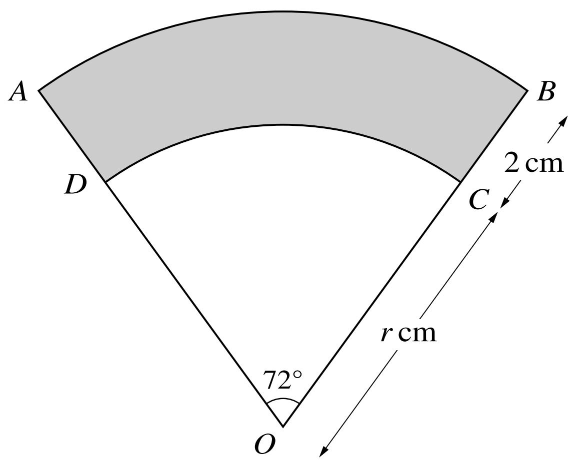
2 Solve the equation $x^{\frac{1}{2}} - 6x^{\frac{1}{4}} + 8 = 0$. [5]

3 A complex number z has modulus 4 and argument $-\frac{1}{3}\pi$. Giving your answers in the form $x + \mathrm{i}y$, find

(a) z, [2]

(b) $\dfrac{8}{\mathrm{i}z}$. [4]

4



The diagram shows a sector AOB of a circle with centre O and radius $(r + 2)$ cm. Angle AOB is 72°. The points C and D lie on OB and OA respectively, and CD is an arc of a circle with centre O and radius r cm. The area of the shaded region $ABCD$ is $8\pi\,\text{cm}^2$.

(a) Express 72° in radians, giving your answer in an exact form. [1]

(b) Determine the value of r. [4]

5 The line L has equation $\mathbf{r} = \begin{pmatrix} 3 \\ 10 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$. The point A has coordinates $(1, -2, 1)$ and the point B has coordinates $(8, 12, -6)$. The line L intersects the line through A and B at the point C.

(a) Find the coordinates of C. [5]

(b) Determine the ratio $AC : CB$, giving the answer in its simplest form. [2]

6 The function f is defined for all real values of x by $f(x) = (x - 14)^3 + 3$.

(a) Find ff(16). [2]

(b) Find $f^{-1}(x)$, stating its domain. [3]

The curve $y = (x - 14)^3 + 3$ intersects the line $y = x$ at the point P.

(c) Use the Newton-Raphson method on $(x - 14)^3 + 3 - x = 0$, with $x_0 = 20$, to find the coordinates of the point P. Show the result of each iteration and give each coordinate correct to 4 significant figures. [3]

(d) State how the point P is related to the curves $y = f(x)$ and $y = f^{-1}(x)$. [1]

7 **(a)** Sketch, on a single diagram, the graphs of $y = \operatorname{cosec} x$ and $y = \tan x$ for $0° \leqslant x \leqslant 360°$. [4]

(b) Solve the equation $\operatorname{cosec} x = \tan x$ for $0° \leqslant x \leqslant 360°$. [5]

8 **(a)** **(i)** Show that $\left(\sqrt{a} - \sqrt{a+b}\right)\left(\sqrt{a} + \sqrt{a+b}\right) = -b$. [1]

(ii) Hence show that $\dfrac{1}{\sqrt{a+b}} - \dfrac{1}{\sqrt{a}} = \dfrac{-b}{a\sqrt{a+b} + \sqrt{a}(a+b)}$. [3]

(b) Given that $f(x) = x^{-\frac{1}{2}}$, use differentiation from first principles to prove that $f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$. [3]

9 Find the solution of the differential equation $\dfrac{dy}{dx} = y\ln(x+2)$, given that $y = 27$ when $x = 1$. Give your answer in the form $f(y) = g(x)$. [7]

10 The expansion of $(1 + ax)^n$ contains the terms $-\frac{2}{3}x$ and $-\frac{112}{81}x^3$.

Find the possible values of a and n. [7]

11 **(a)** Show that $\dfrac{d}{dx}\left(\sec x \tan x + \ln(\sec x + \tan x)\right) = 2\sec^3 x$. [7]

[You may quote the derivatives of $\sec x$ and $\tan x$ from the list of formulae (MF20).]

(b) The region bounded by the curve $y = \dfrac{1}{1 - x^2}$, the x-axis and the lines $x = 0$ and $x = \frac{1}{2}$ is rotated completely around the x-axis. Use the substitution $x = \sin u$ to show that the volume generated is given by $\frac{1}{12}\pi(4 + 3\ln 3)$. [6]

BLANK PAGE